WHIMC Exoplanet Sky Color

Skylight Spectrum of Exoplanets with Terrestrial Atmospheres

The second attempt to determine the sky color of exoplanet scenarios on the WHIMC server. With much appreciated help from Dr. Craig F. Bohren at PennState, who is a colleague of Dr. Neil Comins, a rather simple derivation of sky color based on the temperature of the host star.

Beginning with Planck's law, equation 1, in terms of wavelength and temperature. Where we omit unnecessary multiplicative factors in the original function, is represented as.

1) $$P(\lambda, T) = \frac{1}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

The constants in equation 1 are known to be:

- $k = 1.38065 * 10^{-23}$ [J/K] Boltzmann constant
- $h = 6.62607 * 10^{-34}$[Js] Planck's constant
- $c = 2.99792 * 10^{8}$ [m/s] Speed of light in a vacuum

Reader set variable:

- T = temperature of host star [K]

Next we multiply both sides of equation 1 by the equation for Rayleigh scattering. We know that the equation for Rayleigh scattering is directly proportional to $\frac{1}{\lambda^4}$, for simplicity this is our multiplicative factor for Rayleigh scattering. Equation 2 represents this step in calculation.

2)

$$\frac{P(\lambda, T)}{\lambda^4} = \frac{1}{\lambda^9} \frac{1}{e^{\frac{hc}{\lambda k T}} - 1}$$

To find the peak wavelength of this function, we must take the derivative of this function and set the equation equal to zero. This calculation will allow us to determine a peak scattered wavelength that is dependent on the temperature of the host star of the exoplanet in question. The equation used to determine the peak wavelength of scattered light is following the derivative calculation below.

$$\frac{d}{d\lambda}\left(\frac{P(\lambda, T)}{\lambda^4}\right) = \frac{d}{d\lambda}\left(\frac{1}{\lambda^9}\frac{1}{(e^{\frac{hc}{\lambda kT}}-1)}\right)$$

Now set this function equal to zero and reduce.

$$= -\frac{9}{\lambda^{10}(e^{\frac{hc}{\lambda kT}}-1)} + \frac{che^{\frac{hc}{\lambda kT}}}{\lambda^{11}kT(e^{\frac{hc}{\lambda kT}}-1)^2}$$

$$\frac{9}{\lambda^{10}(e^{\frac{hc}{\lambda kT}}-1)} = \frac{che^{\frac{hc}{\lambda kT}}}{\lambda^{11}kT(e^{\frac{hc}{\lambda kT}}-1)^2}$$

Wavelength λ, is now a maximum denoted λ_m

$$9 = \frac{ch}{\lambda_m kT}\frac{e^{\frac{hc}{\lambda kT}}}{(e^{\frac{hc}{\lambda kT}}-1)}$$

Here, we will now approximate $(e^{\frac{hc}{\lambda kT}}-1) \approx e^{\frac{hc}{\lambda kT}}$ to get

$$9 = \frac{Ch}{\lambda_m kT}$$

$$\lambda_m T = \frac{Ch}{9k}$$ Plug in all known constents to get a value for $\lambda_m T$

To get:

$$\lambda_m T = 1600[\mu m K]$$

The equation for maximum wavelength dependant on star temperature is

3) $\lambda_m T = 1600\ [\mu m K]$

We know the desired wavelength of light for our own Earth's atmosphere is in the blue light range. Using T=6000K for the temperature of the Sun yielding the maximum wavelength value of $\lambda_m = 260nm$. This is in the UV spectrum of light. Fortunately the Ozone in the Earth's atmosphere absorbs the majority of all the UV radiation from the Sun. Moving towards the longer wavelengths of light. Directly after UV light is blue light range, which is as expected since the human eye cannot see UV light.

As another example of this work in action, one can calculate an approximate scattered light wavelength for an exoplanet. One of such exoplanets on the WHIMC server is Trappist-1e. Trappist is presumed to be Earth-like. This exoplanet orbits the ultra cool red dwarf star with a temperature of $T = 2566\ K$. This temperature of star yields a maximum scattered wavelength of $\lambda_m = 624\ nm$ which falls in the yellow-orange light range. We can now convert this wavelength into an RGB value and then a HEX color code, which can be implemented as the sky color on the exoplanet worlds on the WHIMC server.